Rossmoyne Senior High School

MATHEMATICS SPECIALIST UNITS 1&2
Section Two:
Calculator-assumed

If required by your examination administrator, please place your student identification label in this box

WA student number: In figures

Circle your teacher: **Alvaro**   **Chua**   **Koulianos**   **Tanday**

Your name _____

Time allowed for this section
Reading time before commencing work: ten minutes
Working time: one hundred minutes

Number of additional answer booklets used (if applicable):

Materials required/recommended for this section
To be provided by the supervisor
This Question/Answer booklet
Formula sheet (retained from Section One)

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in this examination

Important note to candidates
No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	52	35
Section Two: Calculator-assumed	13	13	100	94	65
				Total	100

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.

3. You must be careful to confine your answers to the specific question asked and to follow any instructions that are specific to a particular question.

4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5. It is recommended that you do not use pencil, except in diagrams.

6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

Markers use only		
Question	Maximum	Mark
9	3	
10	6	
11	7	
12	8	
13	8	
14	8	
15	8	
16	9	
17	8	
18	3	
19	9	
20	9	
21	8	
S2 Total	94	
S2 Wt (×0.6633)	65%	

Section Two: Calculator-assumed   **65% (94 Marks)**

This section has **thirteen** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 100 minutes.

Question 9   **(3 marks)**

Prove that the opposite angles of a cyclic quadrilateral are supplementary.

DO NOT WRITE IN THIS AREA AS IT WILL BE CUT OFF

Question 10   **(6 marks)**

(a)   Triangle ABC has vertices $A(2, -3)$, $B(2, 5)$ and $C(12, -1)$. Determine the area of this
      triangle after it has been transformed using the matrix $\begin{bmatrix} -4 & 4 \\ 3 & 3 \end{bmatrix}$.   (3 marks)

(b)   Show use of matrix algebra, including any inverse matrix used, to solve the following
      system of linear equations: (3 marks)

$$2a + 3b = 55$$
$$4a + 5b = 79$$

Question 11 (7 marks)

(a) In the diagram shown (not to scale) ABC is a straight line and B, C, D and E lie on a circle.

AE is a tangent to the circle at E, $\angle BEC = 76°$ and $\angle BDE = 27°$.
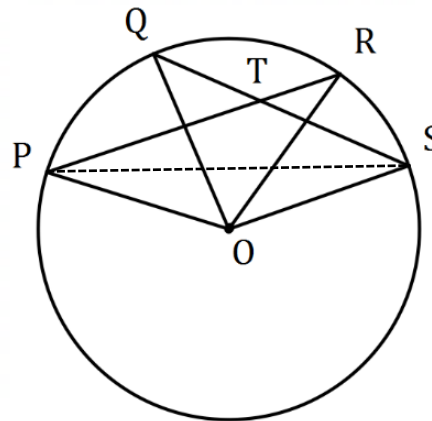
Determine, with reasons, the size of $\angle BAE$.
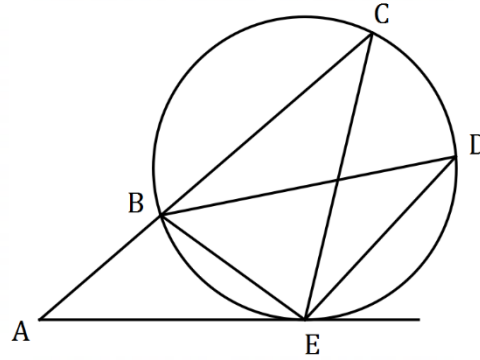
(4 marks)

(b) In the diagram shown (not to scale) P, Q, R and S lie on a circle centre O and chords QS and PR intersect at T.

$\angle POQ = 42°$ and $\angle ROS = 35°$.

Determine, with reasons, the size of $\angle RTS$.

(3 marks)

Question 12 **(8 marks)**

The vertices of triangle T are $A(-4, -3)$, $B(13, 6)$ and $C(-2, 8)$.

Transformation M is a translation by vector $\begin{bmatrix} 7 \\ -6 \end{bmatrix}$.

(a) State the coordinates of the image of A after triangle T is transformed by M. (1 mark)

Transformation N is a reflection in the line $y = x$.

(b) Determine the transformation matrix for N and state the coordinates of the image of B after triangle T is transformed by M and then by N. (3 marks)

Transformation P is a rotation of $45°$ clockwise about the origin.

(c) Determine the exact coordinates of the image of C after triangle T is transformed by N and then by P. (3 marks)

(d) Write a matrix expression for the transformation matrix Q that represents the inverse of transformation N followed by the inverse of transformation P.
Leave your answer in terms of N and P. There is no need to simplify your expression.
(1 mark)

Question 13 **(8 marks)**

Two vectors are $\mathbf{p} = \begin{pmatrix} 72 \\ -154 \end{pmatrix}$ and $\mathbf{q} = \begin{pmatrix} -39 \\ 252 \end{pmatrix}$. Determine

(a) the magnitude of $\mathbf{p}$. (1 mark)

(b) the angle between the directions of $\mathbf{q}$ and $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$. (2 marks)

(c) the value of the scalar constant k so that $18\mathbf{p} + k\mathbf{q}$ is parallel to $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$. (2 marks)

(d) a vector $\mathbf{r}$ that is perpendicular to $\mathbf{p}$ with the magnitude of $\mathbf{q}$. (3 marks)

Question 14 **(8 marks)**

(a) Determine the number of integers between 1 and 749 that are

    (i) divisible by 72. (1 mark)

    (ii) divisible by 8 or by 9 but not by 72. (3 marks)

(b) A playlist offered by a music streaming service has 15 different songs. Every time a playlist is streamed, the songs are shuffled into a random arrangement.

Show that after the playlist has been streamed 75 000 times, at least 3 of those streams began with the same 4 songs in the same order. (4 marks)

See next page

Question 15 (8 marks)

(a) State whether each of the following statements are true or false, supporting each answer with an example or counterexample.

(i) $\forall\, a, b, c, d \in \mathbb{R}$, if $a < b$ and $c < d$ then $ac < bd$. (2 marks)

(ii) $\forall\, n \in \mathbb{N}, n^2 = 2 \times {}^nC_2 + {}^nC_1$. (2 marks)

(b) Prove by contradiction that $ABCD$ is not a cyclic quadrilateral if diagonal AC of length 13 cm cuts diagonal BD of length 12 cm at E so that $AE = DE = 4$ cm. (4 marks)

Question 16   **(9 marks)**

Office buildings are heated during cold winter days by powerful air-conditioning units. Although the temperature of individual offices rises quickly and stays the same during business hours, the actual concrete and metal structure of the building takes time to completely heat and cool again.

During a cold winter day, the air-conditioning system is powered up at $7am$ before all office staff arrive, and then it is turned off later in the afternoon. The temperature $T(t)$ of the office building, in degrees Celsius, for any time t in hours from midnight, can be modelled by a sinusoid according to the rule:

$$T(t) = v - 8\cos(\omega t + \theta)$$

The coldest temperature that the building reaches is $12°C$ and it occurs at $7am$ and $7pm$.

(a)   Determine the value of v, ω and θ, and hence state the function for $T(t)$.   (4 marks)

(b)   Sketch the graph of $T(t)$ on the grid below for the domain $7 \leq t \leq 19\ hrs$.   (3 marks)
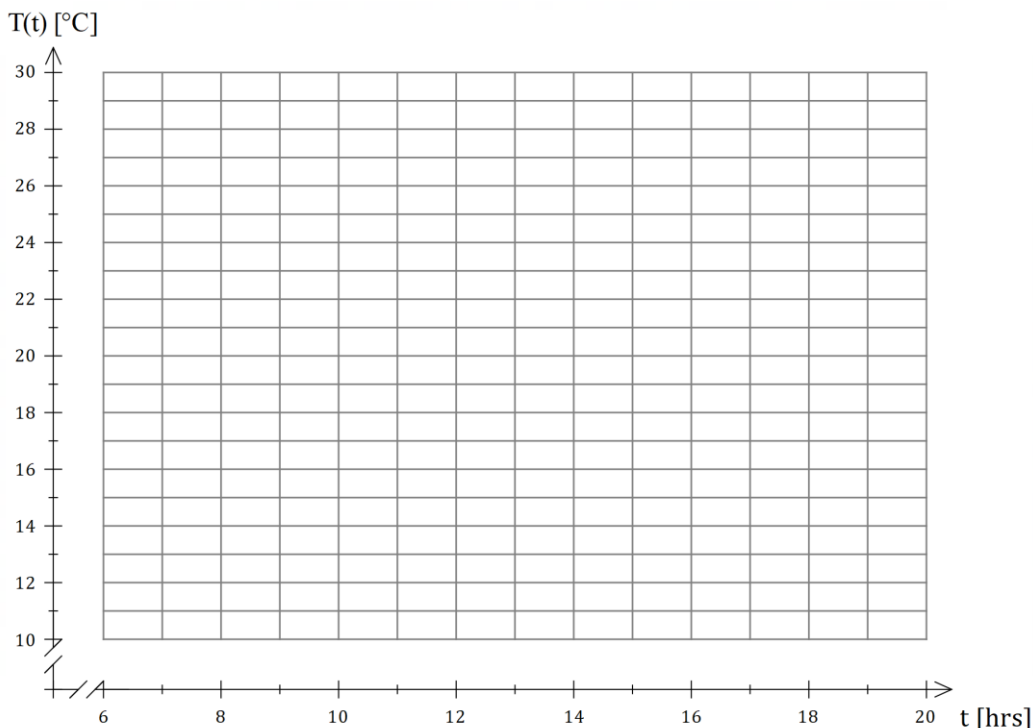
(c)   Determine the proportion of time over the given domain when the temperature of the building is above $24°C$.   (2 marks)

Question 17 **(8 marks)**

(a)   5 students from Class A, 8 from Class B and 10 from Class C have nominated for the 3 places available in the team for a mathematics competition. Determine the number of different teams that can be formed if

   (i)   the students are chosen from the same class. (2 marks)

   (ii)   at least 2 students in the team are chosen from Class A. (2 marks)

(b)   Prove that for $n \geq 5$, $^nC_4 + {}^nC_5 = {}^{n+1}C_5$. (4 marks)

Question 18 **(3 marks)**

Let $B = \begin{bmatrix} -1 & 5 \\ 2 & -8 \end{bmatrix}$ and $C = \begin{bmatrix} 7 \\ -11 \end{bmatrix}$. Determine X when $X - 5BC = B^2X$.

Question 19   **(9 marks)**

(a)   Determine each of the following in terms of n.

     (i) $^nC_1 + {}^nC_2 + {}^nC_3 + \cdots + {}^nC_n \quad =$ (1 mark)

     (ii) $^nC_{n-1} \times {}^{n+1}C_0 \quad =$ (2 marks)

(b)   For each of the following, write expressions using the notation ^nC_r and/or ^nP_r as needed. Do **not** evaluate your answer.

     (i)   How many 5-character passwords can be created from the lower-case letters of the alphabet, without repetition, that contain exactly three vowels?   (2 mark)

     (iii)   How many 8-character passwords can be created from the lower-case letters of the alphabet and the digits 0 to 9, without repetition, that contain exactly three vowels OR exactly three consonants?

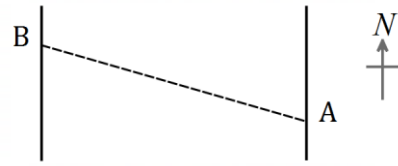   (4 marks)

Question 20 **(9 marks)**

Points A and B lie on opposite sides of a river so that B is 320 m away from A on a bearing of 290°.

A uniform current flows due south in the river between A and B at 0.35 m/s.

Riley can swim at a steady speed of 1.4 m/s and plans to swim from A to B and then back to A.

(a) Determine the bearing Riley should swim to move directly towards B from A. (3 marks)

(b) Show that Riley takes 42 seconds less to swim the return leg than the first leg. (6 marks)

Question 21 **(8 marks)**

A common proof that $\sqrt[4]{3}$ is irrational begins by assuming that $\sqrt[4]{3}$ is rational, so that $\sqrt[4]{3} = \dfrac{a}{b}$.

(a)   Describe two properties of variables a and b that the proof requires, other than $b \neq 0$.

(2 marks)

The next step obtains the relationship $a^4 = 3b^4$, from which it is deduced that $a = 3A, A \in \mathbb{Z}$.

(b)   Prove, using the contrapositive, that if a^4 is a multiple of 3 then so is a.   (4 marks)

(c)   Complete the proof that $\sqrt[4]{3}$ is irrational.   (2 marks)

End of questions

Supplementary page

Question number: _____

Supplementary page

Question number: _____

Supplementary page

Question number: _____